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Perturbative Gluon Exchange in a Covariant Quark Model of the Pion

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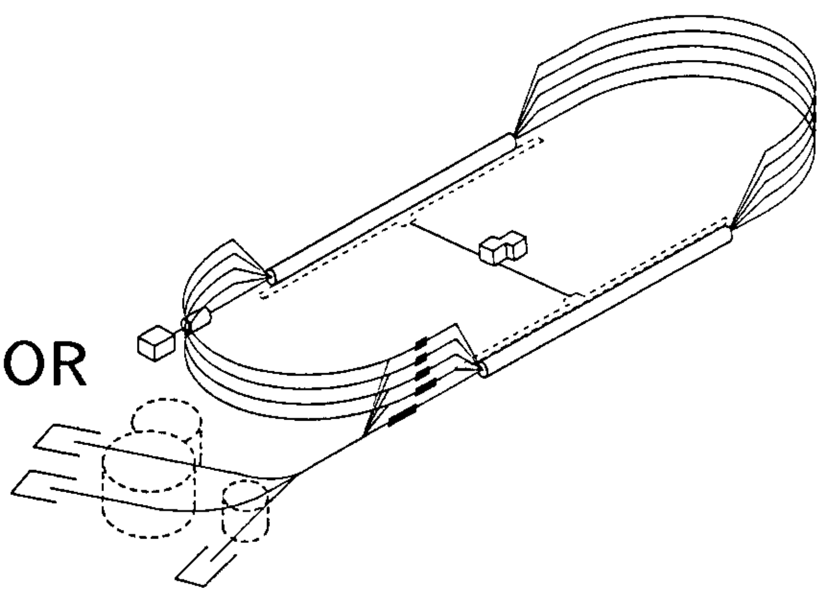

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Abstract

A covariant pion wave function, which reproduces the low energy data, is used to calculate the perturbative gluon exchange contributions to the pion charge form factor. It is found that the perturbative process dominates at $q > 3.5 GeV/c$. The dependence on the quark mass and the asymptotic behavior of the form factor are explicitly displayed.

One well known result of perturbative QCD(PQCD) is that the perturbative one gluon exchange(OGE) diagrams shown in Fig.1 give an exact prediction for the asymptotic form factor of the pion[1,2]. The momentum transfer at which this process actually dominates the form factor is still a matter of some uncertainty, however. Some would argue that the data suggest that the asymptotic result is already dominate at Q^2 as low as 3 GeV^2/c^2[3], while others suggest that the asymptotic result does not dominate until much higher Q^2, at least as large as $15 \sim 20 GeV^2/c^2$[4]. In these approaches[1-4], the OGE process is evaluated in the infinite momentum/light-cone frame or by boosting the nonrelativistic wave function. In this short note we evaluate the OGE contributions to the pion form factor using a "soft" wave function previously obtained from a *covariant* quark model[5], and find that they dominate in the region above $10 \sim 15 GeV^2/c^2$.

In any quark model of the pion, it is assumed that the leading contribution to the pion form factor is given by the impulse diagram shown in Fig.2. If the exact pion wave function were used, the asymptotic result obtained from this diagram would be identical to the OGE contributions of Fig.1. However, for a "soft" quark model wave function(by which we mean a wave function obtained *only* from the confining part of the interaction), the impulse diagram contains *no* contributions from OGE processes, and calculation of the OGE diagrams gives not only an estimate of the PQCD contributions, but also an estimate of the momentum transfer range over which the model can be used without adding OGE contributions.

The invariant amplitude of the gluon exchange process shown in Fig. 1a is given by

$$J_1^\mu = iC_u \int\int \frac{d^4k d^4k'}{(2\pi)^8} Tr\left\{\overline{\Psi}(p',k')\gamma^\mu S(k'+\frac{p'}{2}-q)\gamma^\alpha \Psi(p,k)\gamma^\beta\right\}\Delta_{\alpha\beta}(u), \qquad (1a)$$

where $S(p)$ is the Feynman quark propagator with mass m_q. The amplitude of Fig. 1b is

$$J_2^\mu = iC_u \int\int \frac{d^4k d^4k'}{(2\pi)^8} Tr\left\{\gamma^\beta\overline{\Psi}(p',k')\gamma^\alpha S(k+\frac{p}{2}+q)\gamma^\mu \Psi(p,k)\right\}\Delta_{\alpha\beta}(u), \qquad (1b)$$

where $p(p')$ and $k(k')$ are the initial(final) momentum of the pion and relative momentum of the $q\overline{q}$ pair, respectively. The gluon-quark vertex is $-ig\frac{\lambda^a}{2}\gamma^\mu$, and C_u is given by the matrix elements; $C_u = <\chi_f|(-iQ_q)|\chi_f><\chi_c|\frac{\lambda}{2}\cdot\frac{\lambda}{2}|\chi_c>(-ig)^2 = -\frac{4}{3}ie_u g^2$, where χ_c and χ_f are the wave functions of color and flavor. The explicit form of charge(Q_q) and SU(3)-color(λ^a) matrices are found in ref.[6] (We take π^+ case for definiteness.) The propagator of the gluon is

$$\Delta_{\alpha\beta}(u) = -i\frac{g_{\alpha\beta} - (1-\xi)u_\alpha u_\beta/u^2}{u^2}, \tag{2}$$

with $u = k - k' + \frac{q}{2}$. The coupling constant defined by $\alpha_s = \frac{g^2}{4\pi}$ is given by

$$\alpha_s(u^2) = \frac{4\pi}{\beta\ ln(-u^2/\Lambda^2_{QCD})}, \tag{3}$$

where $\beta = 11 - \frac{2}{3}n_f(n_f = 3)$ and $\Lambda_{QCD} = 150MeV$[4] are used. We approximate the gluon momentum(u) in the coupling constant by the photon momentum as the logarithmic dependence is quite mild. The effect of the QCD gauge fixing(ξ) will be discussed later.

Our soft wave function has the form

$$\Psi(p,k) = \frac{\mathcal{N}}{D(k^2)}\left[A\gamma^5 + B\gamma^5 \not{p}\right]\chi_c\chi_f$$

$$= S(k+\frac{p}{2})\Gamma(p,k)S(k-\frac{p}{2}), \tag{4}$$

where $\Gamma(p,k)$ is the $\pi q\overline{q}$ vertex function. The spin structure of a pseudo-scalar meson is represented by γ^5 and $\gamma^5\not{p}$ with the mixing parameter(η); $A \equiv (1-\eta)$ and $B \equiv \eta/M$ ($Mc^2 = 138MeV$ is used). The momentum dependence is given by $D(k^2) = \prod_{i=1}^{3}[k^2 - \Lambda_i]^2$, and this form allows us to estimate the typical mass scales of physical processes. These parameters are analyzed to fit the charge radius ($r_\pi = 0.66 \pm 0.03 fm$), the weak decay constant($f_\pi = 93 \pm 0.5 MeV$) and the low-q^2 data of the charge form factor(shown in the solid circles in Fig.3) calculated with the impulse diagram(Fig.2). Analysis of the χ^2- fit shows quite stable results for different values of the quark mass(m_q) and yields the result[5]; $\Lambda_1 = 0.74 fm^{-1}$, $\Lambda_2 = 3.2 fm^{-1}$, $\Lambda_3 = 3.7 fm^{-1}$, $\eta = 0.083$, $r_\pi = 0.64 fm$ $f_\pi = 92.8 MeV$ and $\chi^2/data \sim 0.5$ with $m_q c^2 = 100 MeV$. For the other choices of the quark masses($m_q c^2 = 10, 300$, and $400 MeV$) the results are essentially same, and the predictions are indistinguishable in the charge form factor with the impulse diagram. We use these parameters in the present calculation and examine the dependence of the gluon exchange effect on the quark mass.

A little manipulation of the γ- matrices separates $J_1^\mu (J_2^\mu)$ into three terms; $J_1^\mu = J_{1A}^\mu + J_{1B}^\mu + J_{1C}^\mu$ ($J_2^\mu = J_{2A}^\mu + J_{2B}^\mu + J_{2C}^\mu$),
where

$$J_{1A}^\mu = iC_u \mathcal{N}^2 \iint \frac{d^4k d^4k'}{(2\pi)^8} \frac{g_a p'^\mu + g_b p^\mu + f k'^\mu + h_\nu^\mu k'^\nu}{([k' + \frac{p'}{2} - q]^2 - m_q^2) u^2 D(k^2) D(k'^2)}, \tag{5a}$$

$$J_{2A}^\mu = iC_u \mathcal{N}^2 \iint \frac{d^4k d^4k'}{(2\pi)^8} \frac{g_b p'^\mu + g_a p^\mu + f k^\mu - h_\nu^\mu k^\nu}{([k + \frac{p}{2} + q]^2 - m_q^2) u^2 D(k^2) D(k'^2)}, \tag{5b}$$

and

$$J_{1B}^\mu = iC_u \mathcal{N}^2 \iint \frac{d^4k d^4k'}{(2\pi)^8} \frac{8[\xi - 1] m_q AB(p \cdot u) u^\mu}{([k' + \frac{p'}{2} - q]^2 - m_q^2) u^4 D(k^2) D(k'^2)}, \tag{6a}$$

$$J_{2B}^\mu = iC_u \mathcal{N}^2 \iint \frac{d^4k d^4k'}{(2\pi)^8} \frac{8[\xi - 1] m_q AB(p' \cdot u) u^\mu}{([k + \frac{p}{2} + q]^2 - m_q^2) u^4 D(k^2) D(k'^2)}. \tag{6b}$$

The last terms are

$$J^{\mu}_{1C} = iC_u\mathcal{N}^2 \iint \frac{d^4k d^4k'}{(2\pi)^8} \frac{-2[\xi-1]B^2(p\cdot u)Tr\{\not{u}\not{p}'\gamma^{\mu}(\not{k}'+\not{p}'/2-\not{q})\}}{([k'+\frac{p'}{2}-q]^2-m_q^2)u^4D(k^2)D(k'^2)}, \tag{7a}$$

$$J^{\mu}_{2C} = iC_u\mathcal{N}^2 \iint \frac{d^4k d^4k'}{(2\pi)^8} \frac{-2[\xi-1]B^2(p'\cdot u)Tr\{\not{u}(\not{k}+\not{p}/2+\not{q})\gamma^{\mu}\not{p}\}}{([k+\frac{p}{2}+q]^2-m_q^2)u^4D(k^2)D(k'^2)}. \tag{7b}$$

Here g_a, g_b , f and h^{μ}_{ν} are given by

$$g_a = (8A^2+4B^2q^2+16m_qAB)+[\xi-1](2A^2+2B^2q^2+4m_qAB), \tag{8a}$$

$$g_b = (-16A^2+4B^2m_\pi^2-8m_qAB)+[\xi-1](-4A^2+2B^2m_\pi^2-4m_qAB), \tag{8b}$$

$$f = (-16A^2+8B^2[m_\pi^2-\frac{q^2}{2}])+[\xi-1](-4A^2+4B^2[m_\pi^2-\frac{q^2}{2}]), \tag{8c}$$

and

$$h^{\mu}_{\nu} = (8B^2+4B^2[\xi-1])[p_\nu p'^{\mu}-p'_\nu p^{\mu}]. \tag{8d}$$

We note that J^{μ}_{2A} , J^{μ}_{2B} and J^{μ}_{2C} are given by interchanging $p' \leftrightarrow p$ $(q \to -q)$ in the expressions of J^{μ}_{1A} , J^{μ}_{1B} are J^{μ}_{1C} , after interchanging the integral variables(k and k'). Therefore, the total amplitude for the gluon exchange process has the electromagnetic gauge invariant form $[J_1+J_2]^{\mu} = F^{\pi}_{gluon}(q^2)[p'+p]^{\mu}$, with no dependence on $q^{\mu} = [p'-p]^{\mu}$, reflecting the symmetric structure of this exclusive process. The other terms coming from the $\bar{d}$-quark coupling to the photon give a similar structure and are included in this calculation.

The loop diagram of the weak decay process gives the following expression[5] for f_π,

$$f_\pi = \frac{4B\sqrt{n_c}}{\sqrt{2}} \int \frac{d^4k}{(2\pi)^4} \frac{\mathcal{N}}{D(k^2)}. \tag{9}$$

In evaluating the asymptotic limit($q^2 \to \infty$), the rapid convergence of $1/[D(k^2)D(k'^2)]$ in the double loop integrals of eqs.(5),(6) and eq.(7)allows the replacements, $[k' + p'/2 - q]^2 - m_q^2 \to q^2/2$, $[k + p/2 + q]^2 - m_q^2 \to q^2/2$ and $u^2 \to q^2/4$. The exact result for the asymptotic behavior can therefore be expressed in terms of the integral of eq.(9). For the eqs.(5a,b),

$$\begin{aligned} &lim_{q^2\to\infty}[J^\mu_{1A} + J^\mu_{2A}] \\ &= [p' + p]^\mu \frac{8iC_u}{q^4}\mathcal{N}^2 \left[\int \frac{d^4k}{(2\pi)^4}\frac{1}{D(k^2)}\right]^2 \\ &\times \left\{4B^2(q^2+m_\pi^2) - 8A^2 + 8m_qAB + [\xi-1][2B^2(q^2+m_\pi^2) - 2A^2]\right\}, \end{aligned} \tag{10a}$$

and this term, including the $\bar{d}$-quark contribution , gives the following asymptotic result

$$F_A^\pi(Q^2) = \frac{16\pi\alpha_s(Q^2)}{Q^2} f_\pi^2 \times \frac{1}{9}[4 + 2(\xi - 1)], \tag{10b}$$

where $Q^2 = -q^2 \geq 0$. The next term is $F_B^\pi(Q^2) \sim 1/Q^6$ since $lim_{q^2\to\infty}(J^\mu_{1B} + J^\mu_{2B})$ behaves like $\sim 1/q^6$, and finally

$$\begin{aligned} &lim_{q^2\to\infty}[J^\mu_{1C} + J^\mu_{2C}] \\ &= \frac{8iC_u}{q^6}\mathcal{N}^2 \iint \frac{d^4k d^4k'}{(2\pi)^8} \frac{q^4[p' + p]^\mu + O(q^2)}{D(k^2)D(k'^2)} \{-2B^2[\xi - 1]\}. \end{aligned} \tag{11a}$$

This becomes

$$F_C^\pi(Q^2) = \frac{16\pi\alpha_s(Q^2)}{Q^2} f_\pi^2 \times \frac{1}{9}[-2(\xi - 1)]. \tag{11b}$$

Therefore, the asymptotic expression for the total form factor, $(lim_{q^2\to\infty}F^\pi(Q^2) = F_A^\pi(Q^2) + F_B^\pi(Q^2) + F_C^\pi(Q^2))$, becomes

$$F^\pi(Q^2) = \frac{16\pi\alpha_s(Q^2)}{Q^2} f_\pi^2 \times \frac{4}{9}. \tag{12}$$

Observe the absence of any dependence on either the QCD gauge parameter(ξ) or the quark mass. This result is very similar to the one obtained by PQCD $(lim_{Q^2\to\infty}F^\pi(Q^2) = \frac{16\pi\alpha_s(Q^2)}{Q^2} f_\pi^2)$[1,2], and is numerically consistent with the result given by Ref.4. The result is also consistent with simple momentum-power counting.

For the finite range of q^2, the double-loop integrals in eqs.(5),(6) and eq.(7) are calculated by the Monte-Carlo method for Feynman parameter integrals. The numerical results are shown in Fig.3 for each quark mass, along with the contribution from the impulse process. The gluon exchange process starts to dominate over the soft contribution at $q = 3 \sim 4GeV/c$. The dependence on the gauge fixing(ξ) is absent in the asymptotic limit, where the bound quarks are almost free. In principle, any observable should not depend on the gauge of QCD. Any sensitivity to this gauge parameter is therefore a measure of the breakdown of the model, and it is important to know how big this is. The sensitivity is shown in Fig.4, and a comparison with Fig.3 shows that these violations are not numerically large, and become very small at momentum transfers where the PQCD contributions are large.

We summarize this work: (1) The covariant quark model for a $q\bar{q}$ pion wave function, which reproduces the low energy properties, provides an estimation of the perturbative gluon exchange effect. This result is found to be consistent with the light-cone quark model[4]. (2) Precise power counting for the asymptotic form factor shows the absence of dependences on the quark mass and gauge fixing. (3) The numerical evaluation of the gluon exchange process shows the dominance at $q = 3 \sim 4GeV/c$ over the impulse contribution.

Our results suggest that the addition of perturbative OGE processes to those already included in the soft covariant wave function will give reasonable estimates for observables in both low- and high-q^2 regions. It is known, within the nonrelativistic quark cluster model of hadronic interaction[8], that the OGE(in a simple form) gives rise to the short-range repulsion between hadrons. We are motivated by the challenge to develop a covariant model of hadronic quark structure for use in relativistic nuclear many-body dynamics[9,10].

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Figure captions

Fig. 1

The diagrams of gluon exchange processes in the pion charge form factor, where the photon couples to the u-quark. The spiral line stands for the gluon with momentum u, and the wavy line is the photon with momentum q

Fig. 2

The impulse diagram for the pion charge form factor, where the wavy line stands for the photon with momentum q.

Fig. 3

The gluon exchange contribution to the pion charge form factor, with the quark mass $m_q = 100MeV(\diamond)$, $300MeV(\circ)$ and $400MeV$ $(+)$. The solid line without symbol is the result for the impulse calculation, and the data are taken from Ref.[7].

Fig. 4

The gauge dependence of the gluon exchange contribution, where the solid line is the calculation with $\xi = 1$ and the dashed-line with $\xi = 0$. The other symbols are same as in Fig.3.

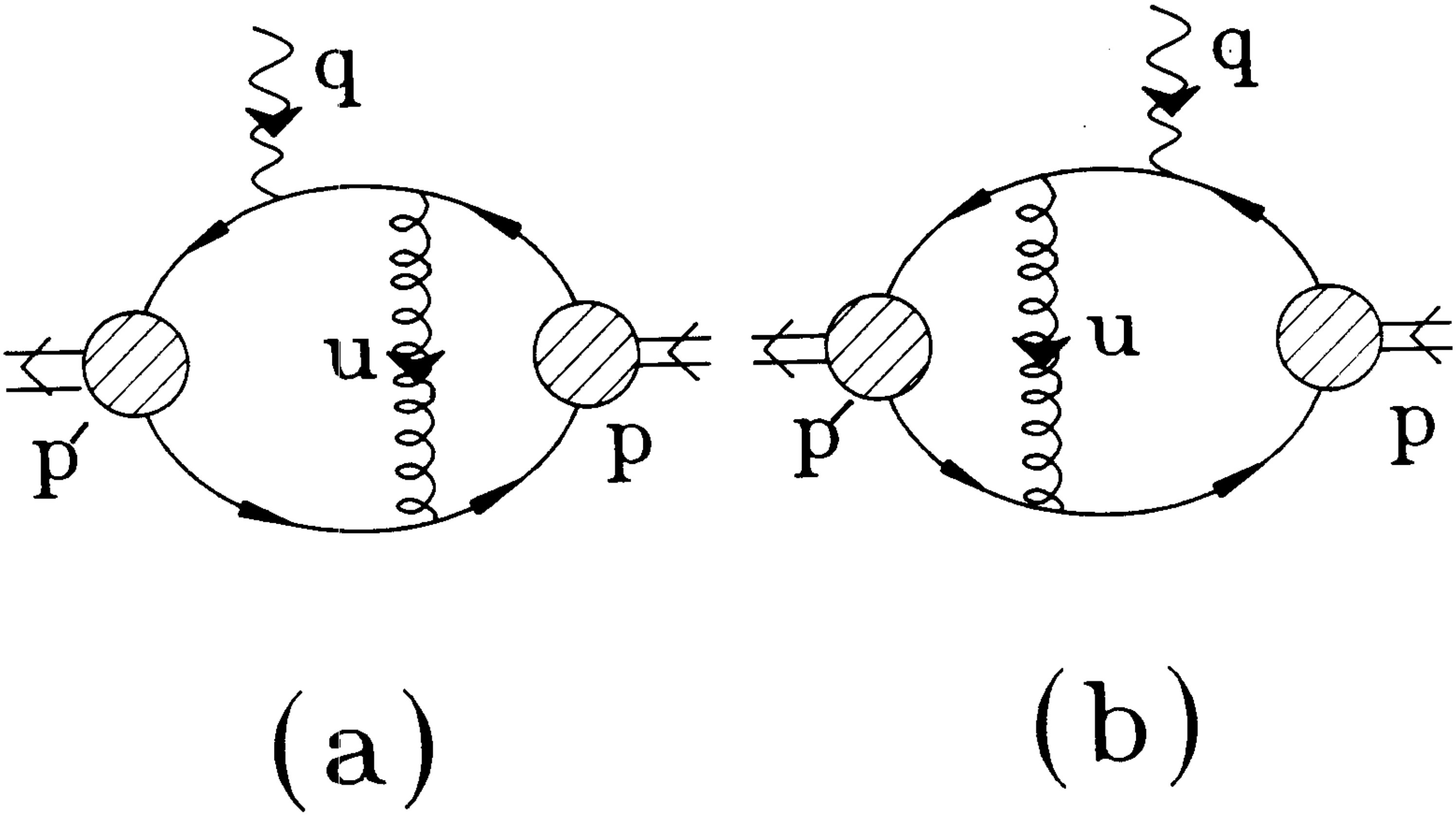


Fig. 1

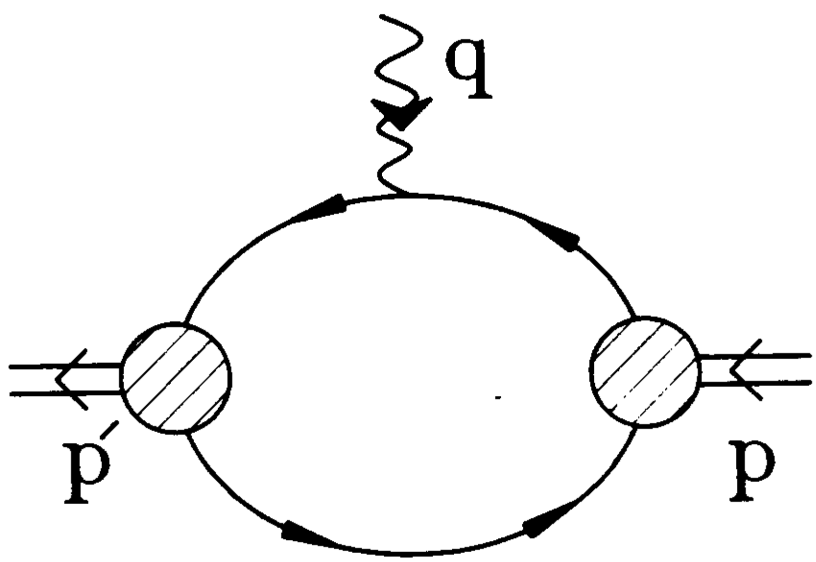
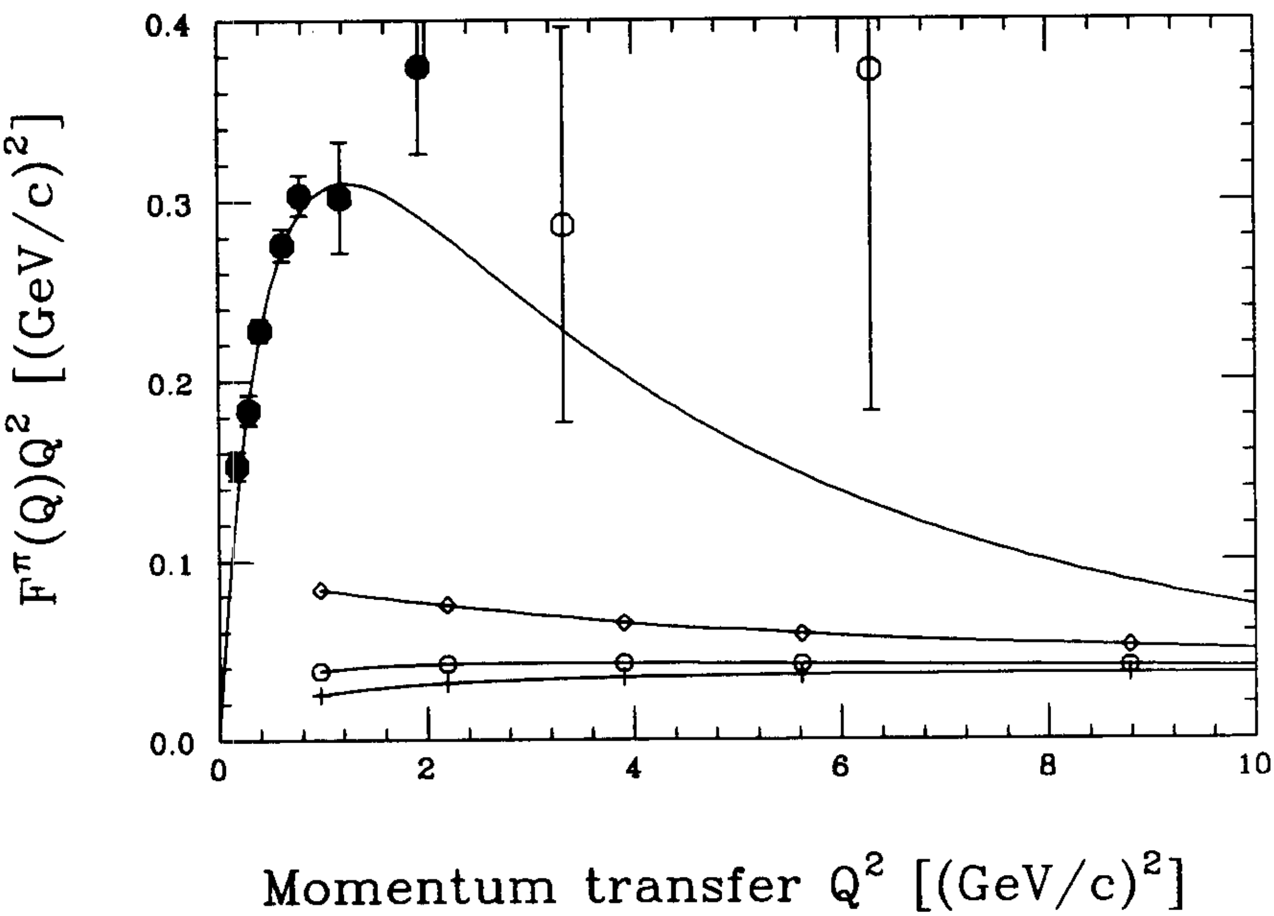


Fig. 2



0.4
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0.2
0.1
0.0
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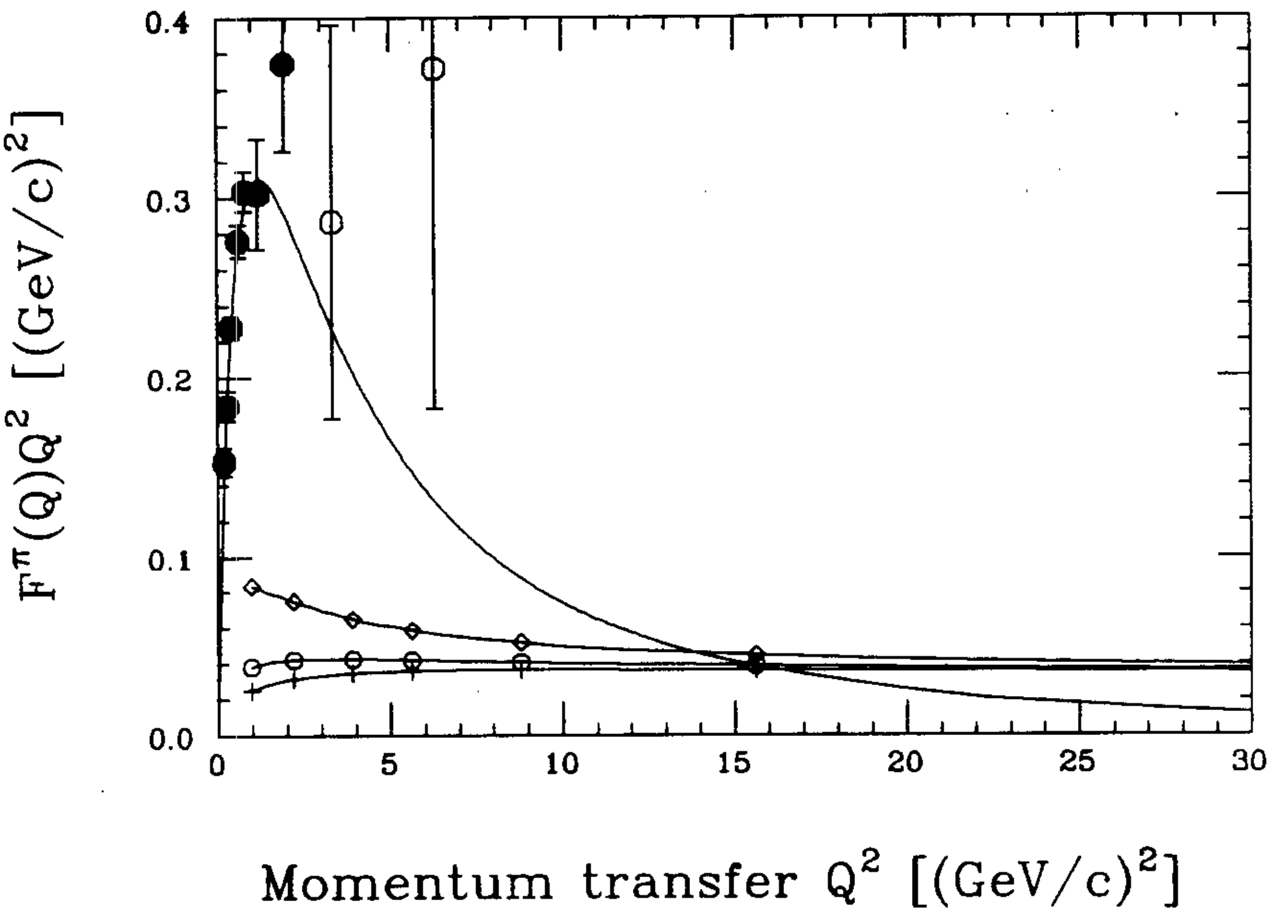
2
4
6
8
10
$F^{\pi}(Q)Q^2$ [(GeV/c)^2]
Momentum transfer Q^2 [(GeV/c)^2]



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5
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15
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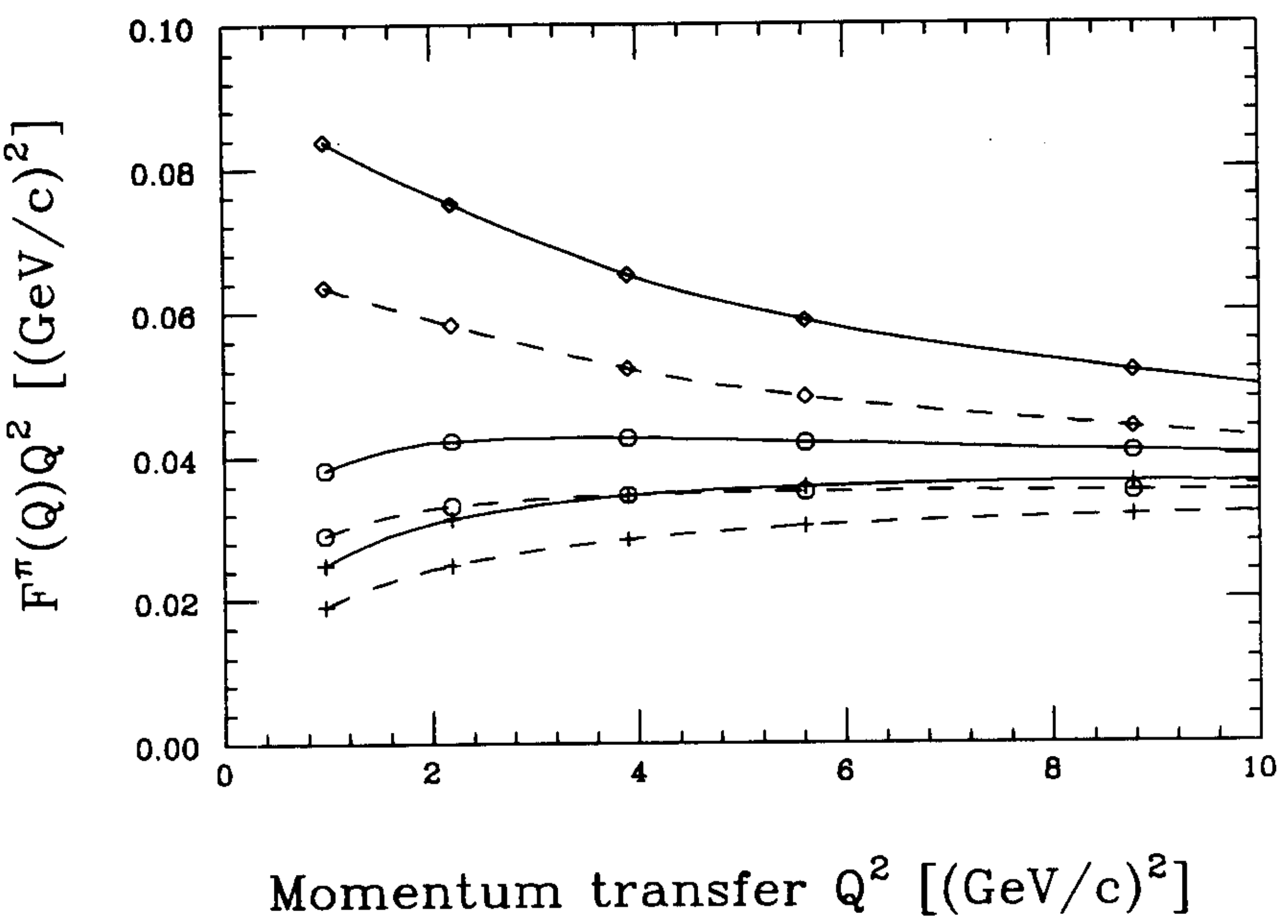
25
30
$F^\pi(Q)Q^2$ [(GeV/c)^2]
Momentum transfer Q^2 [(GeV/c)^2]

Fig. 30



0.10
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0.06
0.04
0.02
0.00
0
2
4
6
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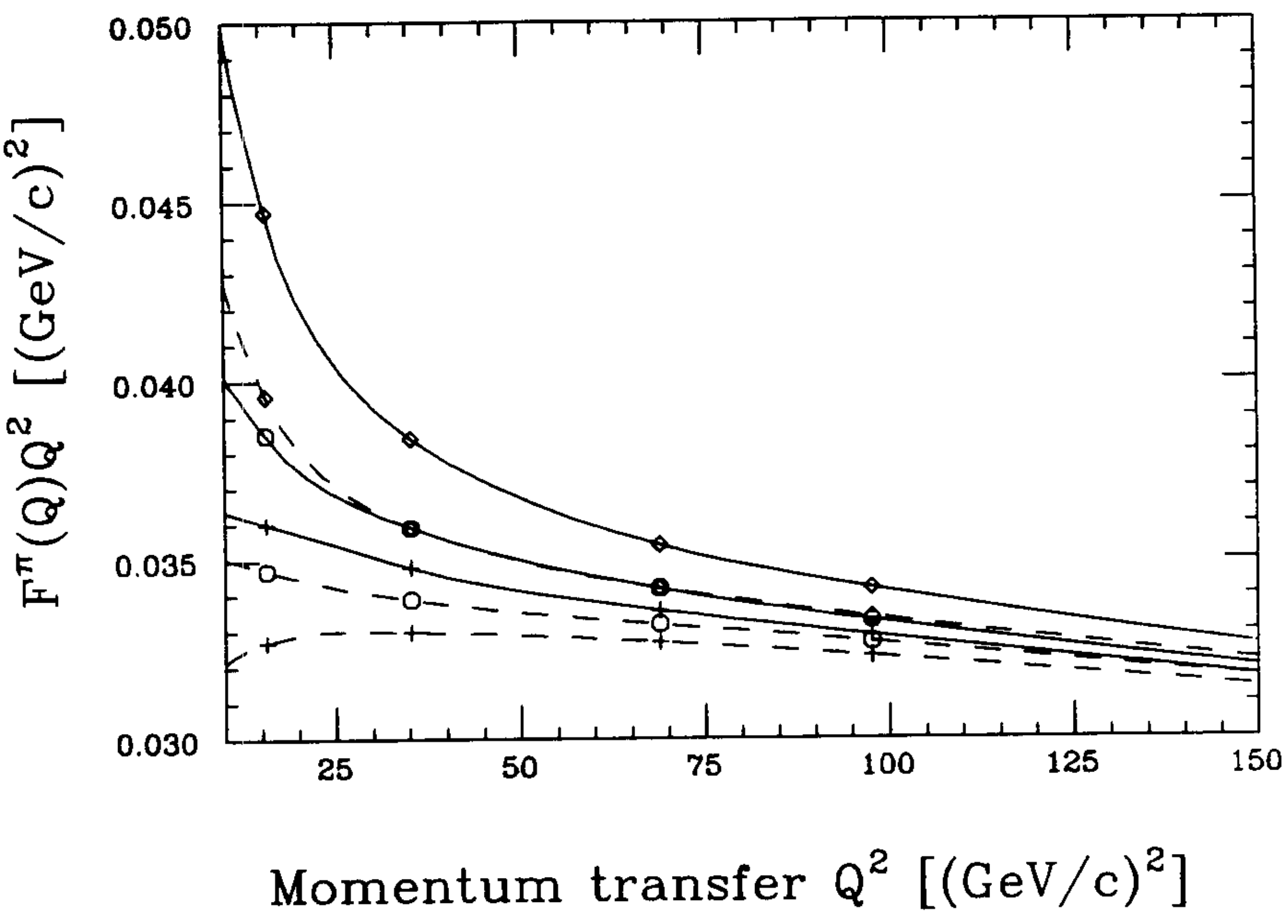
10
$F^{\pi}(Q)Q^2$ [$(GeV/c)^2$]
Momentum transfer Q^2 [$(GeV/c)^2$]

Fig. 4a



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0.035
0.030
25
50
75
100
125
150
$F^{\pi}(Q)Q^2$ [(GeV/c)^2]
Momentum transfer Q^2 [(GeV/c)^2]

Fig. 4b